Regression Diagnostics and Troubleshooting

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Question

How do regression diagnostics fit into analysis?

Steps in Regression

- For any model
  1. Run regression
  2. Check for departures from CLR assumptions
  3. Attempt to fix those problems
- Additionally, compare between models based on purpose, fit, and diagnostics

OLS assumptions

1. Linearity $y = X\beta + \varepsilon$
2. Iid sample $y_i, x_i')$ iid sample
3. No perfect collinearity X has full rank
4. Zero conditional mean $E(\varepsilon|X) =)$
5. Homoskedasticity $Var(\varepsilon|X) = \sigma^2 I_N$
6. Normality $\varepsilon|X \sim N(0, \sigma^2 I_N)$

- 1-4: unbiased and consistent β
- 1-5: asymptotic inference, BLUE
- 1-6: small sample inference

OLS Problems

1. Perfect collinearity: Cannot estimate OLS
2. Non-linearity: Biased β
3. Omitted variable bias: Biased β.
4. Correlated errors: Wrong SEs
5. Heteroskedasticity: Wrong SEs
6. Non-normality: Wrong SEs - p-values.
7. Outliers: Depends on where they come from

Topics for Today

1. Omitted Variable Bias
2. Measurement Error
3. Non-Normal Errors
4. Missing data

Omitted Variable Bias: Description

- The population is

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \varepsilon_i$$

- But we estimate a regression without X_2

$$y_i = \hat{\beta}_0 + \hat{\beta}_1^{(omit)} x_{1,i} + \varepsilon_i$$

Omitted Variable Bias: Problem

Coefficient Bias

$$E\left(\hat{\beta}_1^{(omit)}\right) = \beta_1 + \beta_2 \frac{\text{Cov}(X_2, X_1)}{\text{Var}(X_1)}$$

Bias Components

- β_2: Effect of omitted variable X_2 on Y
- $\frac{\text{Cov}(X_2,X_1)}{\text{Var}(X_1)}$: Association between X_2 and X_1

Omitted Variable Bias: Hueristic Diagnostic

- Heuristic: sensitivity of the coefficient to inclusion of controls
- If insensitive to inclusion of controls, OVB less plausible
- Note: sensitivity of **coefficient** not p-value.

"These controls do not change the coefficient estimates meaningfully, and the stability of the estimates from columns 4 through 7 suggests that controlling for the model and age of the car accounts for most of the relevant selection." (Lacetera et al. 2012)

Omitted Variable Bias: Diagnosing Statistic

- Suppose X and Z observed, and W unobserved in,

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \varepsilon$$

- Statistic to assess importance of OVB

$$\delta = \frac{\text{Cov}(X, \beta_3 W)}{\text{Cov}(X, \beta_2 Z)} = \frac{\hat{\beta}_C}{\hat{\beta}_{NC} - \hat{\beta}_C}$$

- If Z representative of all controls, then large δ implies OVB implausible
- Example in Nunn and Wantchekon (2011)

Omitted Variable Bias: Reasoning about Bias

If know omitted variable, may be able to reason about its effect

$\text{Cov}(X_1, X_2)$	$\text{Cov}(X_2, Y) > 0$	$\text{Cov}(X_2, Y) = 0$	$\text{Cov}(X_2, Y) < 0$
> 0	$+$	0	-
0	0	0	0
< 0	-	0	$+$

Omitted Variable Bias: Solutions by Design

- OVB always a problem with methods relying on selection on observables
- Other methods (Matching, propensity scores) may be less model dependent, but still can have OVB
- Preference for methods relying on identification in other ways
  - experiments
  - instrumental variables
  - regression discontinuity
  - fixed effects/diff-in-diff

Measurement Error in X: Description

- We want to estimate

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

- But we estimate

$$Y_i = \beta_0 + \beta_1 X_1^* + \beta_2 X_2 + \epsilon$$

- Where X_1^* is X_1 with measurement error

$$X_i^* = X_i + \delta$$

where $\mathrm{E}(delta) = 0$, and $\mathrm{Var}(\delta) = \sigma_\delta$.

Measurement Error in X: Problem

- Similar to OVB
- For variable with the measurement error
  - $\hat{\beta}_1$ biased towards zero (**attenuation bias**)
- For other variables:
  - $\hat{\beta}_2$ biased towards OVB bias.
  - When measurement error high, it's as if that variable is not controlled for

Measurement error in Y

- Population is
$$Y_i = \beta_0 + \beta_1 X_{1,i} + \epsilon$$
- But we estimate
$$Y_i + \delta_i = \beta_0 + \beta_1 X_{1,i} + \varepsilon_i$$
- β not biased, but larger standard errors
$$Y_i = \beta_0 + \beta_1 X_{1,i} + (\epsilon_i + \delta_i)$$
where $\mathrm{E}(\epsilon_i + \delta_i) = 0$, and $\mathrm{Var}(\varepsilon_i + \delta_i) = \sigma^2_\varepsilon + \sigma^2_\delta$.
- If each δ_i has different variances, then heteroskedasticity

Measurement Error: Solutions

- If in treatment variable:
  - get better measure
- If in control variables:
  - include multiple measures. Multicollinearity less problematic than measurement error.
- Models for measurement error: Instrumental variables, structural equation models, Bayesian models, multiple imputation.

Non-Normal Errors

- Usually not-problematic
- Does not bias coefficients
- Only affects standard errors, only for small samples
- **But** may indicate
  - Model mis-specified
  - $E(Y|X)$ is not a good summary
- Diagnose: QQ-plot of (Studentized) residuals

Missing Data in X

Listwise Deletion

- Drop row with *any* missing values in Y or X
- Problem: If missingness correlated with X, coefficients biased

Multiple Imputation

- Predict missing values from non-missing data
- Multiple imputation packages: **Amelia**, **mice**
- Almost always better than listwise deletion

More complicated Missing Data Problems

- MNAR: Missing not-at randrom in X.
  - Values in X do not predict missingness
  - Need to model the selection process
- Truncation or censored dependent variable: specific MLE models